Solution by PROFESSOR F. L. GRIFFIN, Williams College.

However the ellipse is placed relative to the co-ordinate axes, its area is

$$S=\int_{x_1}^{x_2}(y''-y')\,dx,$$

where x_1 and x_2 $[x_1<x_2]$ are the extreme abscissas taken in the curve, and y' and y'' $[y'\leqq y'']$ are the two ordinates corresponding to any one value of x. Solved for y, the given equation becomes

$$by=-(hx+f)\pm\sqrt{[f^2-bc+2(hf-bg)x+(h^2-ab)x^2]}.$$

Now let

$$f^2-bc=b^2A,\quad fh-bg=b^2B,\quad ab-h^2=b^2C,$$

where for an ellipse $C>0$. Then the difference of the two ordinates becomes

$$y''-y'=2\sqrt{[A+2Bx-Cx^2]}.$$

Hence, integrating,

$$S=2\left[\frac{Cx-B}{2C}\sqrt{[A+2Bx-Cx^2]}+\frac{B^2+AC}{2C^{\frac{3}{2}}}\sin^{-1}\frac{Cx-B}{\sqrt{[B^2+AC]}}\right]_{x_1}^{x_2}.$$

Now the extreme abscissas make $y'=y''$, or $A+2Bx-Cx^2=0$; whence

$$Cx_2=B+\sqrt{[B^2+AC]}\ \text{ and }\ Cx_1=B-\sqrt{[B^2+AC]}.$$

Substituting these values,

$$S=\frac{B^2+AC}{C^{\frac{3}{2}}}[\sin^{-1}1-\sin^{-1}(-1)]=\frac{\pi}{b}\,\frac{[(fh-bg)^2+(ab-h^2)(f^2-bc)]}{(ab-h^2)^{\frac{3}{2}}},$$

which immediately reduces to the formula proposed.

Also solved by G. B. M. Zerr and J. Scheffer.

MECHANICS.

240. Proposed by S. A. COREY, Hiteman, Iowa.

A perfectly flexible wire rope weighing one pound per foot is suspended from the tops of two vertical supports 300 feet apart, one support being 30 feet higher than the other. One end of the rope is fastened to the top of the higher support, while 600 feet of the rope hangs vertically from the top of the lower support. Assuming that the rope is free to slide over the top of the lower support without friction, find the lowest point of

that portion of the rope which is suspended between the supports. Also find the amount of work which must be performed in raising the lowest point to make it coincide with the top of the lower support by exerting a pull on the free end of the rope.

[No solution of this problem has been received.]

240. Proposed by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

A simple beam length $2a$, supported at both ends, is loaded in the form of a parabola, height of vertex b. Find deflection at center due to this load.

Solution by the PROPOSER.

Let AB be the beam, ACD the parabola, $CD=b$, $AD=DB=a$. Take E any point on AB, draw EF perpendicular to AB. Let $AE=x$, and also z be the distance of the center of gravity of the area AEF from EF. Then $\frac{4}{3}ab$=total load. $(x-a)^2+(a^2/b)(y-b)=0$ is the equation to the parabola, with A as origin.

$$\text{Then } (x-z)=\frac{\int\int xdxdy}{\int\int dxdy}=\frac{\int\int xdxdy}{A}.$$

$$\therefore A(x-z)=\int_0^x xydx=\frac{b}{a^2}\int_0^x(2ax^2-x^3)dx=\frac{bx^3}{12a^2}(8a-3x).$$

$$A=\int_0^x ydx=\frac{bx^2}{3a^2}(3a-x).$$

$$\therefore x-z=\frac{8ax-3x^2}{4(3a-x)} \text{ and } z=\frac{4ax-x^2}{4\,(3a-x)}.$$

Taking moments about A we get,

$$EI\frac{d^2y}{dx^2}=\tfrac{2}{3}abx-Az=\tfrac{2}{3}abx-\frac{bx^3}{3a}+\frac{bx^4}{12a^2}=M.$$

$$EI\frac{dy}{dx}=\tfrac{1}{3}abx^2-\frac{bx^4}{12a}+\frac{bx^5}{60a^2}+C.$$

When $x=a$, $dy/dx=0$, $C=-\frac{4}{15}a^3b$.

$$\therefore EI\frac{dy}{dx}=\tfrac{1}{3}abx^2-\frac{bx^4}{12a}+\frac{bx^5}{60a^2}-\tfrac{4}{15}a^3b,$$

$$Ely=\tfrac{1}{9}abx^3-\frac{bx^5}{60a}+\frac{bx^6}{360a^2}-\tfrac{4}{15}a^3bx=-\frac{61a^4b}{360} \text{ when } x=a.$$